

HELIOS-CONSIDERATIONS
FOR THE PRODUCTION AND CONTAINMENT
OF A HIGH-TEMPERATURE PLASMA ⁺)

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The HELIOS project is discussed in some more detail. By comparing the diverse collision times of the plasma particles it follows that the E.C.R. plasma, which is to be created at the first stage of the experiment, must have an electron temperature of at least 200 eV and a neutral gas density lower than 10^{10} particles per cm^3 . For the subsequent heating of the ions a small gradient in the magnetic field and/or a relatively high particle density (which requires a high frequency of the microwaves) is also wanted. The plasma will be stabilized against the flute instability by application of a hexapole field. Microinstabilities are not expected to be very dangerous, partly because of the high electron temperature, partly because of stabilization by the minimum B field.

⁺) Seminar held at the Institut für
Plasmaphysik on February 16, 1965.

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1. Principle

Formation of a high electron temperature plasma by electron
IPP 2/44 reson F. Boeschoten HeliOS-Considerations for the
magnetic mirror field. Subsequent Production and Containment of
cyclotron resonance (I.C.R.) a High-Temperature Plasma with
minimum B fields. Eventual adiabatic (in English).
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Electron and ion temperature at the successive
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 $n_e \approx 10^{12} \text{ cm}^{-3}$.

1. Principle

Formation of a high electron temperature plasma by electron cyclotron resonance (E.C.R.) heating of electrons in a magnetic mirror field. Subsequent heating of the ions with ion cyclotron resonance (I.C.R.) heating. Stable confinement with minimum B fields. Eventual adiabatic compression of the hot plasma¹). The name HELIOS stands for "Heizung von Elektronen und Ionen mit Sender".

2. Heating Scheme

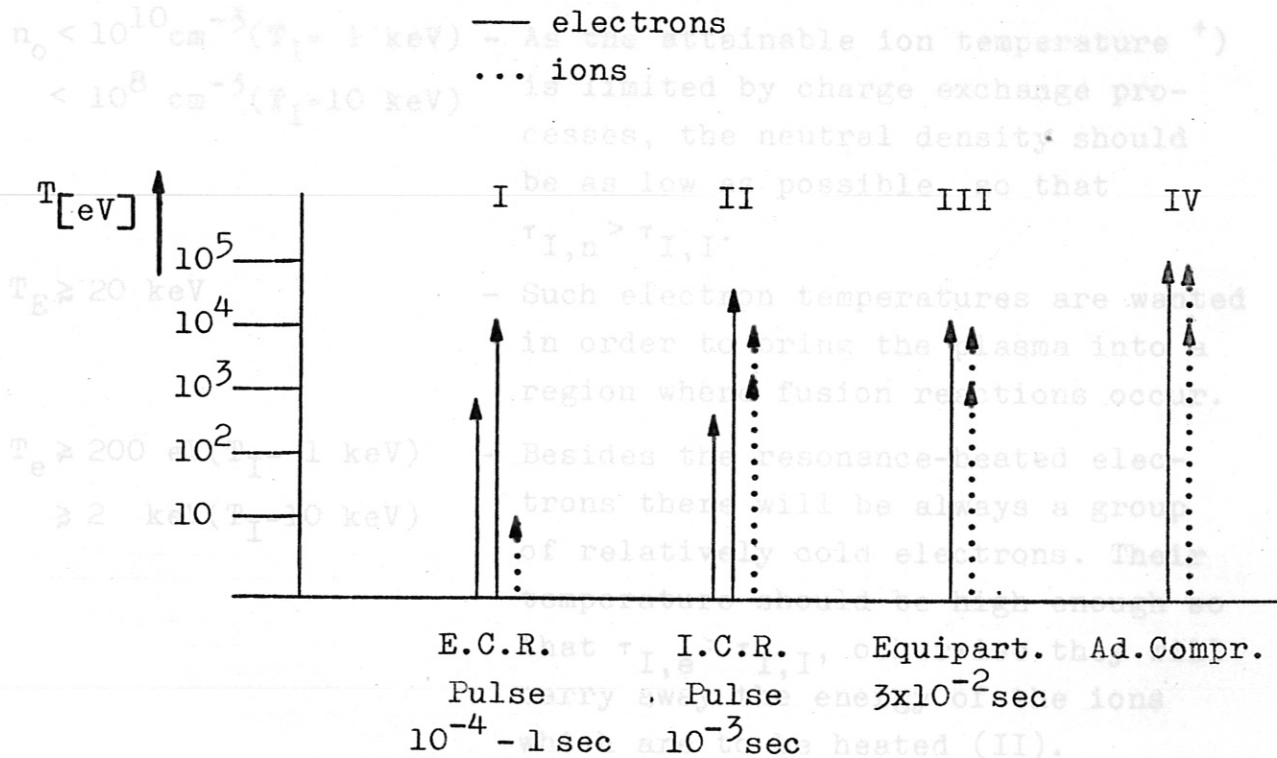


Figure 1

Electron and ion temperature at the successive stages of heating for a plasma with density $n_e \approx 10^{12} \text{ cm}^{-3}$.

¹) The subscripts used are:
I for ions of high temperature, i for ions of low temp.;
E for electrons of high temp., e for electrons of low temp.;
n for neutrals, n_0 for neutral particle density.

I. Production of an E.C.R. plasma with the following parameters:

$$n_e \approx 10^{12} \text{ cm}^{-3}$$

- For the I.C.R. heating it would be more profitable to work with higher densities (see section 5), but at this time microwave generation at very high power level keeps within reasonable costs limits only up to 10 GHz ($n_e \approx 8 \times 10^{11} \text{ cm}^{-3}$) or 20 GHz (expected $n_e \approx 3 \times 10^{12} \text{ cm}^{-3}$). We hope to be allowed to order 20 GHz equipment.

$$n_0 < 10^{10} \text{ cm}^{-3} (T_I = 1 \text{ keV})$$

$$< 10^8 \text{ cm}^{-3} (T_I = 10 \text{ keV})$$

- As the attainable ion temperature ⁺) is limited by charge exchange processes, the neutral density should be as low as possible, so that

$$\tau_{I,n} > \tau_{I,I}$$

$$T_e \geq 20 \text{ keV}$$

- Such electron temperatures are wanted in order to bring the plasma into a region where fusion reactions occur.

$$T_e \geq 200 \text{ eV} (T_I = 1 \text{ keV})$$

$$\geq 2 \text{ keV} (T_I = 10 \text{ keV})$$

- Besides the resonance-heated electrons there will be always a group of relatively cold electrons. Their temperature should be high enough so that $\tau_{I,e} > \tau_{I,I}$, otherwise they will carry away the energy of the ions which are to be heated (II).

Collision times and consequences ($T_e = 200 \text{ eV}$):

$$\tau_{i,i} \approx 10^{-5} \text{ sec, about equal to the confinement time } \tau_{\text{conf}} \text{ of the ions;}$$

$$\tau_{i,e} \approx 4 \times 10^{-2} \text{ sec}$$

$$\tau_{i,E} \approx 40 \text{ sec}$$

$$T_i \text{ keeps low as } \tau_{\text{conf}} \approx \tau_{i,i} \ll \tau_{i,e} \ll \tau_{i,E}$$

$$\tau_{e,e} \approx 6 \times 10^{-5} \text{ sec}$$

$$\tau_{E,E} \approx 6 \times 10^{-2} \text{ sec}$$

$$\left. \begin{array}{l} \tau_{e,e} \approx 6 \times 10^{-5} \text{ sec} \\ \tau_{E,E} \approx 6 \times 10^{-2} \text{ sec} \end{array} \right\} \begin{array}{l} \text{electrons escape together with ions; no ambi-} \\ \text{polar confinement by hot electrons, as cold} \\ \text{electrons compensate the space charge;} \end{array}$$

⁺) The subscripts used are:

I for ions of high temperature, i for ions of low temp.;
E for electrons of high temp., e for electrons of low temp.;
n for neutrals, n₀ for neutral particle density.

$\tau_{e,E} \approx 3 \times 10^{-2}$ sec, no time for energy equipartition of the two groups of electrons.

The ions stay cold (presumably about 10 eV), as the energy transfer of electrons to ions is very insufficient and the ions are scattered rapidly out of the mirror, together with the cold electrons.

II. I.C.R. heating of the ions:

If we assume that the (cold) electrons are hot enough, a limit posed on the achievable T_I is caused by charge exchange ($\tau_{I,n} > \tau_{I,I}$). The required neutral particle density is $n_0 < 10^{10} \text{ cm}^{-3}$ for $T_I = 1 \text{ keV}$ and is $n_0 < 10^8 \text{ cm}^{-3}$ for $T_I = 10 \text{ keV}$. (This is not an absolute limit; if enough power is available, these temperatures may be reached even if $\tau_{I,n} < \tau_{I,I}$.) After heating of the ions the ion-ion collision time becomes $\tau_{I,I} \approx 3 \times 10^{-2} \text{ sec}$ (1 keV) to $\approx 1 \text{ sec}$ (10 keV), whereas the electron-electron collision times and the ion-electron collision times remain practically unchanged. As assumed under I., the ions do not lose their energy to the cold electrons ($\tau_{I,e} \geq \tau_{I,I}$) before they escape from the magnetic bottle. The cold electrons are, however, confined by the ions by ambipolar space charges.

III. Energy equipartition between the two groups of electrons:

As now $\tau_{I,I} \approx \tau_{e,E}$, there is time for the cold electrons to share the energy with the hot electrons, and the result is only one group of electrons with an energy of about 10 keV. Depending on the neutral particle density $T_I = 1$ to 10 keV.

Figure 2

IV. An adiabatic compression of a factor 10 in the magnetic field will yield a plasma with $n_e \approx 10^{13} \text{ cm}^{-3}$, $T_I \approx 10$ to 100 keV, and $T_E \approx 100 \text{ keV}$. Such a compression may be easily performed, as the time required for compression of this hot plasma is 0.1 to 1 sec. For this case even the possibility of using superconducting coils may be considered.

later on (section 5) the gradient in the magnetic mirror field should not be too large, so that the distance between the mirrors should be about 150 cm. The volume thus obtained is about

3. Dimensions and Design

A schematic drawing of the apparatus - made for the case of 10 GHz u.h.f. power - is given in Fig. 2. The u.h.f. energy is fed perpendicular into the plasma as in the Oak Ridge experiments²⁾. The r.f. energy is fed into the plasma from concentric electrodes as in the Berkeley experiment³⁾. As an alternative the r.f. energy could be fed azimuthally into the plasma as in the Princeton experiments on I.C.R. heating⁴⁾. The r.f. energy must then be coupled to the plasma with a Stix coil. However, insulation problems seem more difficult to solve for the inductive coupling than for the capacitive coupling.

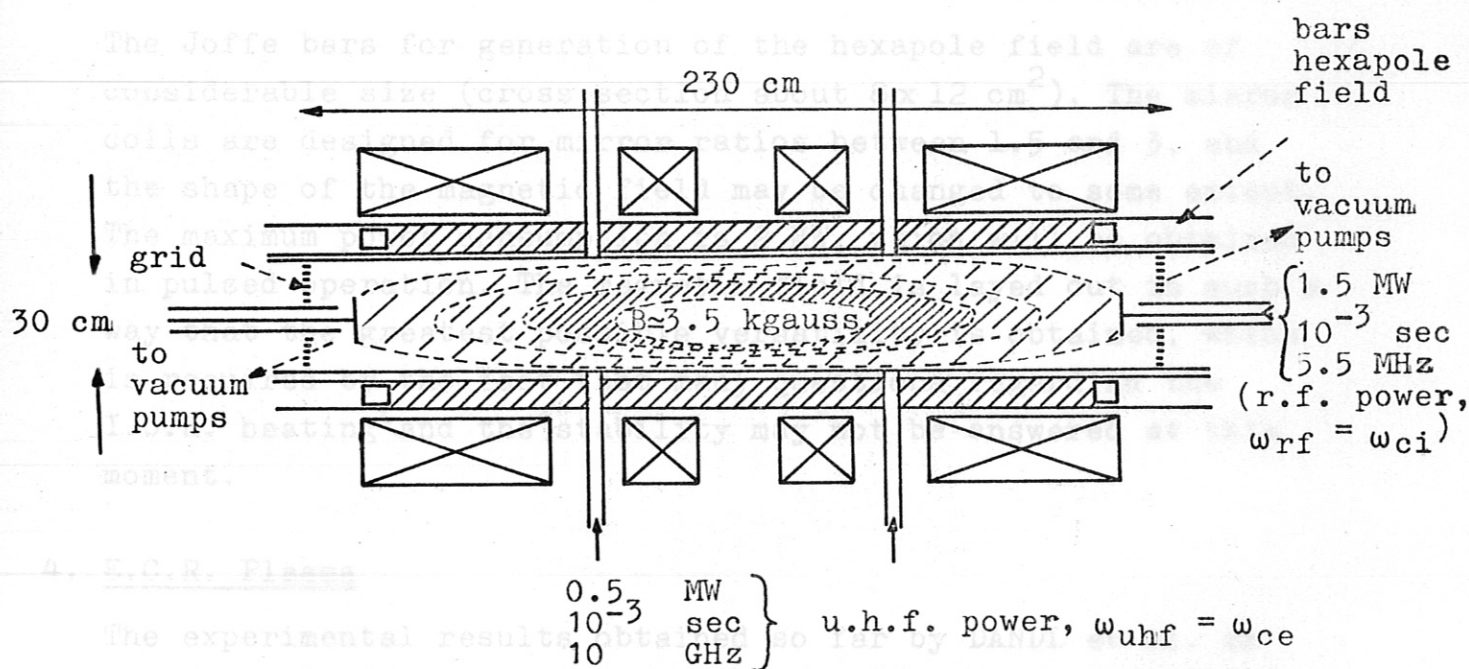


Figure 2

If the experiments are to be made with 10 GHz microwaves, the magnetic field strength corresponds to $B \approx 3.5$ kgauss in the middle of the machine. The Larmor radius of 10 keV hydrogen ions is then about 4 cm. This requires an inner diameter of the vacuum chamber of at least 30 cm. For reasons described later on (section 5) the gradient in the magnetic mirrorfield should not be too large, so that the distance between the mirrors should be about 150 cm. The volume thus obtained is about

150 e. As it serves as a container for the microwave energy, the end surfaces must be made out of grids in order to make evacuation of the vacuum vessel possible. If the E.C.R. plasma contains 100 keV electrons of about 10^{12} cm^{-3} density, as it was found in Oak Ridge, we need about 2 kWsec (= $nkT \times \text{vol.}$) of microwave energy in order to heat the hot electrons. In pulsed operation with a pulse length of about 10^{-3} sec (see section 4) some MW of microwave power are required. Our r.f. transmitter is adjusted to a frequency of 5.5 MHz, where I.C.R. is expected to occur for H^+ ions. The 1.5 MW power during a pulse length of 10^{-3} sec is large enough for heating the ions to the desired temperature. The easiest way of coupling the r.f. energy to the plasma seems to be capacitive, for which purpose two electrodes are immersed in the plasma chamber.

The Joffe bars for generation of the hexapole field are of considerable size (cross section about $8 \times 12 \text{ cm}^2$). The mirror coils are designed for mirror ratios between 1.5 and 3, and the shape of the magnetic field may be changed to some extent. The maximum power consumption is 2 MW, which will be obtained in pulsed operation. The magnetic field is layed out in such a way that the greatest possible versatility is obtained, which is required by the fact that many questions regarding the I.C.R. heating and the stability may not be answered at this moment.

4. E.C.R. Plasma

The experimental results obtained so far by DANDL et al. in Oak Ridge indicate the following: T_E independent of frequency ($\approx 100 \text{ keV}$), independent of u.h.f. power; n dependent on frequency ($\approx 10^{12} \text{ cm}^{-3}$ at 10 GHz), slightly dependent on u.h.f. power; plasma volume proportional to u.h.f. power. Calculations on E.C.R. plasmas are made by SEIDL⁵⁾ and by WIMMEL and WÖHLER⁶⁾. A complete understanding is difficult to obtain because of the complexity of the problem.

In the first instance one could think of the following simple model:

the energy losses of the electrons must be taken into account, if T_e is to be calculated. (Such calculations are done by

The plasma density builds up to such values that $\omega_{pe} \approx \omega_{uhf}$; at higher densities the plasma becomes opaque for the microwaves. Thus $n_e \sim \omega_{uhf}^2$, which is roughly found in experiment. Electrons heated by resonance may acquire unlimited energy, until relativistic effects come into operation at about 100 keV. Many electrons are not resonance-heated, however. This may in part be caused by the fact that the resonance zones occupy only a small part of the volume.

The fact that the resonance-heated electrons are found to acquire much higher energies than required for the heating scheme of HELIOS does not seem to be too bad. It is likely that with better understanding of E.C.R. plasmas a mode of operation will be found such that $T_e \approx 20$ keV. Moreover, the 100 keV electrons are a nuisance because of X-ray radiation, but not an unsurmountable obstacle for I.C.R. heating of the ions.

Much bigger difficulties may be expected from the cold electrons. For the operation of HELIOS it is necessary that they have energies of 200 eV or higher. The following simple argument indicates that this should be possible:

Treating the electrons in the E.C.R. plasma as a system of forced oscillations under friction ⁷), one finds for the energy gain (in erg) of an electron per sec:

$$I \approx \frac{(eE)^2 v_{e,e}}{m_e v_f^2}.$$

If the electrons reach an equilibrium energy such that $I \approx T_e$, we see the following: For a reasonable value, $E = 100$ V/cm, and a microwave power pulse of 1 sec duration, $T_e \approx 500$ eV. For an arbitrary pulse time t and an electrical field E , we find $T_e \sim E^{4/5} t^{2/5}$ or $T_e \sim p^{2/5} t^{2/5}$ (p is the microwave power). Thus T_e is proportional to the $2/5^{\text{th}}$ power of the energy fed into the container, and with a shorter pulse length a correspondingly higher microwave power would be required to reach a certain value of T_e . It is clear that this model is too simple and that the energy losses of the electrons must be taken into account, if T_e is to be calculated. (Such calculations are done by

WIMMEL and WÖHLER.) As $v_f^2 \sim n$, the absorbed energy is independent of v_f . T_e is expected to depend only on the power and not on the frequency of the microwaves.

In addition to the requirements for the plasma parameters (n , T_E , and T_e) there is an upper limit to the neutral particle density n_0 in order to avoid impermissible charge exchange (see section 2).

This requirement is not typical for HELIOS, but for all mirror machines. If we want $\tau_{i,n} \geq \tau_{i,i}$, it means $n_0 \lesssim 10^{-2} n_i / T_i^2$ (T_i in keV), which cannot be obtained easily if T_i is to be 1 to 10 keV.

Before understanding of E.C.R. plasmas is more complete, we can make again a simple estimation. The mean free path length for ionization of neutral particles by electrons is $\lambda_{ion,E} \approx 2$ cm in the n, T_E region under consideration. Thus 10 cm from the wall, n_0 may be diminished by a factor 100. If we want $n_0 \lesssim 10^{10} \text{ cm}^{-3}$ in a region of 10 cm diameter around the center of the machine, the neutral gas pressure at the wall should be less than 0.3×10^{-4} torr. This may not be too difficult to attain, as the discharge is started with pressures of some 10^{-5} torr and pressures of 0.8 to 1.5×10^{-4} torr are measured experimentally at the wall during operation. However, if $n_0 \lesssim 10^8 \text{ cm}^{-3}$ is wanted for heating the ions to 10 keV, the pressure at the wall should be less than 0.3×10^{-6} torr. This seems only possible in pulsed operation with shutting down of the gas feed. Here may be expected considerable difficulties, as the experiments in Oak Ridge have shown that the E.C.R. plasma becomes unstable, if the gas feed is stopped and the neutral density drops. This behavior could be explained, if a cold plasma acts as a surface blanket to the hot plasma and most of the incoming neutrals are ionized in this blanket. After turn-off this blanket is no longer sustained, so that it is lost rather quickly and the plasma becomes unstable. A CRUCIAL POINT IN THE HELIOS PROPOSAL IS THE HOPE THAT THE HOT ELECTRON PLASMA MAY BE STABILIZED AGAINST THIS INSTABILITY WITH A MINIMUM B FIELD, SO THAT THE NEUTRALS ARE NO LONGER NECESSARY FOR STABILIZATION ⁺).

⁺) In a recent publication by the Princeton group on E.C.R. heating is mentioned the possibility of a flute instability being responsible for the unstable plasma losses.

5. I.C.R. Heating

Till now the ion temperatures reached with I.C.R. heating were limited to some 100 eV. It is very promising that the r.f. energy could be fed into the plasma and was supplied to the ions. However, the ions in a short time lost a large part of their energy to the cold electrons (opinion of the Princeton group working on I.C.R. heating ⁺) or perhaps because of some instability (opinion of the Kharkov group working on I.C.R. heating ⁺⁺) ⁸). It may be expected that the E.C.R. plasma is much more suitable for I.C.R. heating than the plasmas used so far. As was shown before, the electrons may probably be heated high enough so that the ions do not lose their energy to them.

The dispersion relation for a cold, collision-free, homogeneous, cylindrical plasma column surrounded by conducting walls is given by STIX ⁹) and by GOULD ¹⁰).

$$k_{\parallel}^2 = \frac{\omega^2}{v_A^2(1 - \frac{\omega^2}{\omega_{ci}^2})} - \frac{1}{2} x_1^2 \pm \sqrt{(\frac{1}{2} x_1^2)^2 + (\frac{\omega^2}{v_A^2(1 - \frac{\omega^2}{\omega_{ci}^2})} \frac{\omega}{\omega_{ci}})^2} ;$$

$x_1 R = \alpha_n$, where α_n is the n^{th} root of the Bessel function I_1 .

The general trend of the dispersion curves is shown in Fig. 3, and was calculated by SILLER for the case $n = 10^{12} \text{ cm}^{-3}$ and $x_1 R = \alpha = 3.84$, $R = 12.5 \text{ cm}$. It may be seen that for $\nu_f = 5.5 \text{ MHz}$ and $B = 3.5$ to 7 kgauss the ion cyclotron waves have a wavelength varying from 2.5 m in the mirror region to 0 at the absorption region in the middle of the machine.

⁺) In recent experiments with the B 66-mirror machine by HOOKE et al. and with the C-stellarator by YOSHIKAWA et al., ion temperatures in the keV range were measured. The electron temperature - about 10 eV - was so low that also in this case the ions lost their energy mainly to the cold electrons.

⁺⁺) In a recent publication of the Kharkov group, the possibility of a flute instability is mentioned. If this is the case, the minimum B field may also prevent instability which is possibly present in I.C.R. experiments.

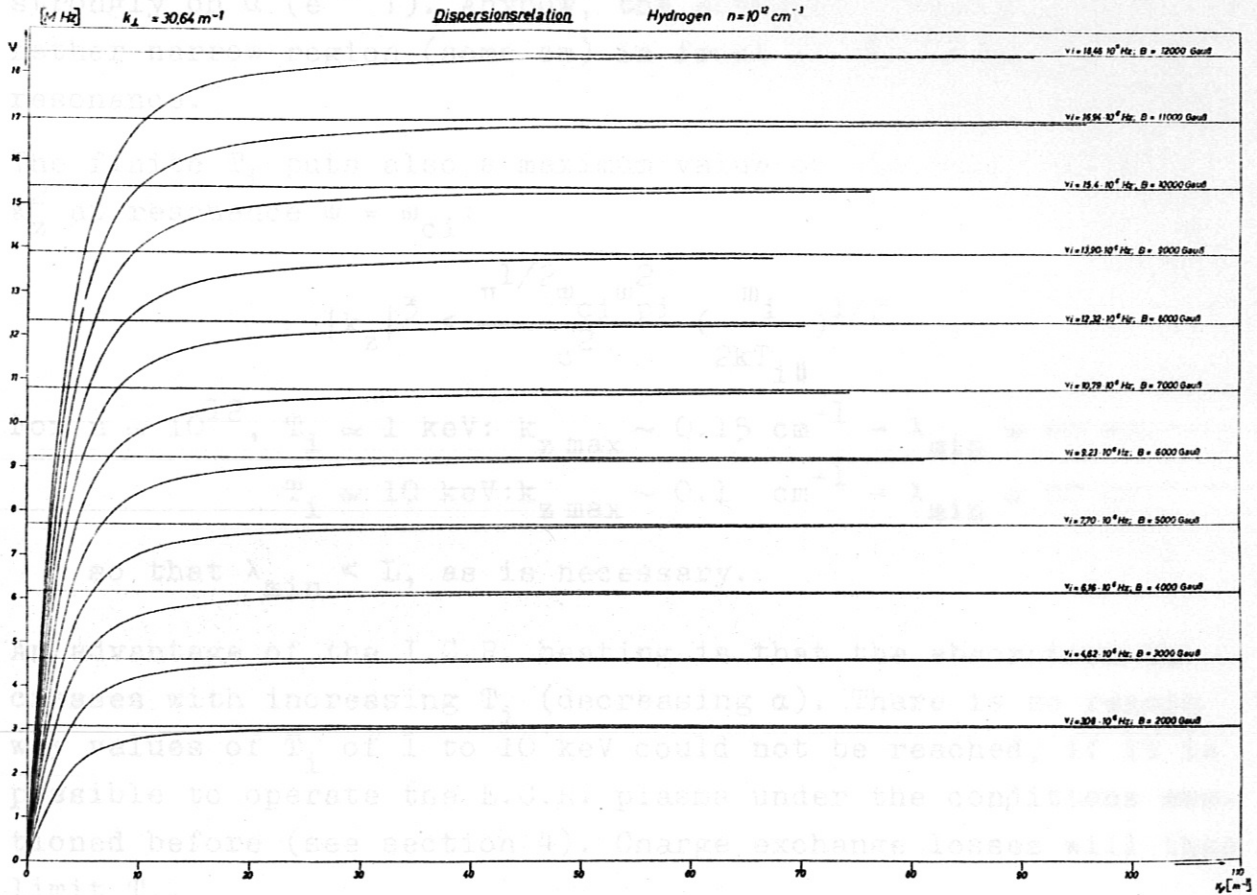


Figure 3

Absorption of the ion cyclotron waves occurs when there are ions that experience a wave frequency equal to their gyrofrequency $\omega_{iz} = (\omega - \omega_{ci})/k_z$. Under the assumption that ω is real and $k = k_r + ik_i$ with $|k_i| \ll |k_r|$, the damping length in z -direction may be estimated from the expression (ref. 9, page 195):

$$\left(\frac{k_i}{k}\right)_z \approx \frac{\omega}{v_{ith}(k_z)} \left(\frac{\omega_{pi}}{k_z c}\right)^2 e^{-\alpha^2}, \text{ where } \alpha^2 = \frac{m_i(\omega - \omega_{ci})^2}{2kT_i k_z^2}.$$

This result is only valid for $\alpha^2 \gg 1$ and $T_{i\perp} = T_{i\parallel}$, which is certainly not true in the resonance region itself. Calculations of the actual absorption are very complicated, as α changes along the mirror field and the absorption depends very

strongly on α ($e^{-\alpha^2}$). Anyhow, the absorption will occur in a rather narrow region (some cm) in front of the place of exact resonance.

The finite T_i puts also a maximum value on the magnitude of k_z^2 at resonance $\omega = \omega_{ci}$:

$$|k_z|^3 < \frac{\pi^{1/2} \omega_{ci}^2 \omega_{pi}^2}{c^2} \left(\frac{m_i}{2kT_{i||}} \right)^{1/2}.$$

For $n \approx 10^{12}$, $T_i \approx 1$ keV: $k_{z \max} \sim 0.15 \text{ cm}^{-1} \rightarrow \lambda_{\min} \approx 40 \text{ cm}$;

$T_i \approx 10$ keV: $k_{z \max} \sim 0.1 \text{ cm}^{-1} \rightarrow \lambda_{\min} \approx 60 \text{ cm}$;

so that $\lambda_{\min} < L$, as is necessary.

An advantage of the I.C.R. heating is that the absorption increases with increasing T_i (decreasing α). There is no reason why values of T_i of 1 to 10 keV could not be reached, if it is possible to operate the E.C.R. plasma under the conditions mentioned before (see section 4). Charge exchange losses will then limit T_i .

Apart from a complication of the calculation of the absorption, the variation of B along the axis causes a minimum value of n_e , which must be present in order that the I.C. waves penetrate the plasma, starting from the electrodes. This values of n_e is approximately given by the condition:

$$k_z \gg \frac{\nabla B}{B}$$

Numerical calculations of $n_{e \min}$ were made recently by HAMILTON¹¹⁾ for the case of a plasma ($n_e \approx 2 \times 10^{13}$, $T_i = 25$ to 50 eV, $T_e = 10$ to 20 eV) contained in a copper cylinder 19.8 cm in diameter and 94 cm long. The magnetic mirror field corresponds to $\nabla B/B \approx 10^{-2} \text{ cm}^{-1}$. Under these conditions it is found that the reflection coefficient $> 1/2$, if $n_e < 10^{13} \text{ cm}^{-3}$. Though these calculations are not strictly valid for HELIOS, these numbers may be used for a rough estimation of the required density. According to the dispersion relation for a cold plasma, k_z varies approximately $\sim n_e^{1/2}$, so that according to HAMILTON's calculations, densities of at least $1/4 \times 10^{13} \text{ cm}^{-3}$ are required for

a machine of 2 m length [†]). ¹⁶⁾ is expected

It is clear that the existing theories on which our estimations are based are not strictly valid for HELIOS. First of all the assumption $T_{i\perp} = T_{i\parallel}$ is certainly not fulfilled. The fact that T_i is not directly affected by the heating mechanism leads one to suspect, however, that HAMILTON's calculations for a low-temperature plasma may be applied approximately. Calculations by GRAWE ¹³⁾ have shown that the expected high T_e values (finite ∇p) do not bring noticeable changes in the results obtained for a cold plasma. The complicated shape of the min. B field will have some influence on the calculations, but this is not expected to be very important. The fact that the plasma density is not constant in the direction along the axis causes some doubts in the validity of the theoretical estimations which we made so far. The whole problem of the coupling of the I.C. waves to the plasma seems to be so complex that only an experiment can bring the answers to our questions. Another complication may arise from the fact that the coherent ion motion is considerable, and it is, therefore, possible that the I.C. mechanism of heating may enhance the development of instabilities. This aspect will be discussed in the next section.

6. Stability

For the evaluation of this integral we should know more. HELIOS is layed out with a hexapole field superposed on the mirror field. It has been shown both experimentally ¹⁴⁾ and theoretically ¹⁵⁾ that the min. B field which is obtained in such a way is stable against the interchange or flute instability. It is possible that the min. B field provides also stabilization against some microinstabilities, like mirror and universal instability. One of the main objects of the HELIOS program will be to make stability studies of a hot plasma with and without min. B field. We will give here a short discussion of the different instabilities which may be encountered in HELIOS.

[†]) Calculations by FILIYA ¹²⁾ from which he concludes that at low temperatures the reflection coefficient is independent of plasma temperature and density and is equal to 1/3, seem less reliable because of the assumptions made.

The interchange instability¹⁶⁾ is expected in those regions in which the field lines bend concavely towards the plasma. In these regions the magnetic field strength decreases in radial direction ($\nabla \times B = 0$) and a lower energy state is achieved, if the plasma interchanges place with magnetic field lines and moves radially out of the high field region. In its simplest form the criterion for interchange instability is:

$$-\int \frac{dl}{RrB^2} > 0, \text{ stable.}$$

Since by definition the curvature R of the magnetic field B is negative at the ends and positive in the middle, the middle section promotes instability. The fact that B is large at the ends will make the middle region dominant.

The validity of this criterion, if applied to mirror machines, is questionable, because:

- 1) It is derived with hydromagnetic equations, whereas the plasma is essentially collision-free.
- 2) The scalar pressure p is assumed constant along a flux line ($B \cdot \nabla p = 0$). If like in HELIOS $p_{\parallel} \neq p_{\perp}$ the criterion becomes

$$-\int \frac{dl(p_{\parallel} + p_{\perp})}{RrB^2} > 0, \text{ stable.}$$

For the evaluation of this integral we should know more about the plasma.

- 3) The boundary conditions which may apply at the ends of the machine, outside the mirrors, have not been taken into account. The behavior of the plasma may be strongly affected by the flow of charges along the magnetic field lines. If like in HELIOS conducting end plates are located at each mirror, the interchange of plasma and magnetic field can no longer take place without disturbance of the field itself (the field lines are 'frozen' into the plasma). This stabilizing effect may be weakened again by the existence of sheaths. As the role played by sheaths is not known, no instability criterion can be given for this case.

Finite Larmor radius stabilization is predicted from microscopic theory. If the plasma is surrounded by a conductor,

the criterion becomes approximately:

$$\frac{L}{r_{pl}^2} \frac{r_{ci}}{C} > C, \text{ stable,}$$

where C is a constant (≈ 1) which depends on geometrical factors and on the plasma density, r_{ci} is the cyclotron radius of the ions and r_{pl} is the radius of the plasma.

For HELIOS ($L = 1.5$ m, $r_{pl} \approx 10$ cm, $B = 3500$ gauss) this criterion becomes $6 \times 10^{-2} \sqrt{T_i [\text{eV}]} = 1$ cm, so that F.L.R. stabilization is expected for $T_i \gtrsim 300$ eV.

Recent experiments in the U.S.S.R., the U.S.A., and France,¹⁷⁾,¹⁸⁾, and ¹⁹⁾, with low β plasmas have shown that the flute instability may be prevented if the magnetic field lines bend convexly towards the plasma ('min. B field') which may be obtained by adding 'Joffe bars' to the magnetic mirror coils. Application of a stabilizing field had in all cases a strong stabilizing effect and all evidence of rapid transport of plasma to the walls (which is observed without stabilizing field) disappeared. As expected, the stabilization sets in if the wall-mirror ratio $\frac{|B|_{\text{wall}}}{|B|_{\text{center}}} \gtrsim 1$. The observed confinement time is determined by charge exchange, so that one may hope to arrive at a collisional diffusion-determined confinement time after improving the degree of ionization. For us the most instructive experiment made so far is the one made by PERKINS and BARR¹⁸⁾ which had comparable plasma data and a hexapole field ($n_e \approx 10^{10} \text{ cm}^{-3}$; $T_e = 4$ to 20 keV; $T_i \approx 1$ keV; $B \leq 10$ kgauss; $p_0 > 10^{-9}$ torr). Since the first experiments were made by Joffe et al., the properties of these so-called 'minimum B' fields have been studied extensively theoretically. For special equilibria of high β plasmas, TAYLOR and HASTIE²⁰⁾ found that the min. B field still provides stabilization against the interchange instability, if:

$$B < B_c \approx \frac{B_1^2 - B_2^2}{B_1^2},$$

where B_1 is the value of $|B|$ on the largest closed $|B|$ contour and B_2 is the actual minimum of $|B|$ in the containment region. B_c can easily attain values as high as $1/3$ or more in practical systems, so that the min. B is still effective for such a high

B plasma as is expected in HELIOS.

Application of a min. B field is of extra importance in HELIOS, because it must not only provide stabilization of the final plasma, but also of the E.C.R. plasma during the build-up phase.

Microinstabilities. It is very difficult to judge the role which microinstabilities may play. So far there is hardly any experimental evidence for those microinstabilities which may occur in HELIOS, and in several cases there is no agreement between the theories. In HELIOS the plasma is created by feeding perpendicular energy to the electrons and the ions. So we have to be aware especially of those instabilities which are due to pressure anisotropy, and we do not expect to be bothered by those which are caused by electrons drifting along the magnetic lines of force. Therefore, we will discuss only the mirror instability, the ion cyclotron overstabilities due to pressure anisotropy, the electrostatic resonance instability, and the universal instability (see also ref. 21).

- a) The mirror instability⁹⁾ is an unstable magnetosonic wave ($\omega < \omega_{ci}$) which may appear under the condition $v_{e\parallel} \gg v_A$, if

$$\frac{\beta_{\perp}^2}{\beta_{\parallel}} > 1 + \beta_{\perp}.$$

For HELIOS: $\beta_{\perp} = 0.1$ to 0.3 ; $T_{e\perp}/T_{e\parallel} \gg 1$ and $T_{i\perp}/T_{i\parallel} \gg 1$;
 $B \geq 3.5 \times 10^3$ gauss and $T_{e\parallel}$ possibly some keV;
 $n_e \approx 10^{12} \text{ cm}^{-3}$.

We find for $v_A \approx 10^9 \text{ cm/sec}$ and $v_{e\parallel}$ some 10^9 cm/sec , so that the condition $v_{e\parallel} \gg v_A$ is just about fulfilled. The instability criterion reduces to $\beta_{\perp} \geq T_{\parallel}/T_{\perp}$ and is probably fulfilled. So it may be possible that the plasma is unstable against the mirror instability. It is, however, reassuring that this instability has never been observed experimentally so far and moreover, that it is possible that it is stabilized by the min. B field.

- b) The cyclotron overstability due to pressure anisotropy⁹⁾ is an unstable I.C. wave (slow Alfvén wave instability) or an unstable E.C. wave. It may appear if $T_{\perp} > T_{\parallel}$ and its

growth rate is given by: $\omega_i \approx \sqrt{\beta_\perp} e^{-1/\beta_\perp} \omega_c$. It is not clear what happens to these instabilities if the plasma is heated exactly at those frequencies where they are expected to occur.

For HELIOS $\beta_\perp \leq 0.3$, so that the slow Alfvén wave instability would have a growth rate of $< 1/50 \omega_c$, which is rather slow. They do not seem to be a serious threat for the stability.

- c) The electrostatic resonance instability (Harris or cyclotron instability) is an electrostatic wave which propagates obliquely to the magnetic field and which may be induced by an anisotropy in the particle distribution function. A resonance builds up between the ion cyclotron frequency and the longitudinal electron plasma oscillations. The most unstable waves have a wavelength which is about equal to the ion Larmor radius ($\lambda_\perp \approx \pi r_{ci}$).

There is no agreement between the different theories. According to DNESTROVSKY, KOSTOMAROV, and PISTUNOVICH²²), instability - for a plasma with hot electrons - occurs if $k_z v_{i\parallel} \ll n \omega_{ci}$ ($k_z \geq 2\pi/L$) and the criteria $\omega_{pe}/\omega_{ci} > 1/(1+\gamma)$ and $T_{i\perp}/T_{i\parallel} > 1 + \frac{1}{\gamma}$ ($\gamma = [(T_{i\parallel}/T_e)(m_e/m_i)]^{1/2}$ and < 1) are fulfilled. Thus it is expected that increasing of electron temperature improves the stability of the plasma.

The condition $k_z v_{i\parallel} \ll n \omega_{ci}$ is fulfilled in HELIOS if one takes $k_z = 2\pi/L$. The stability of the system will improve if L decreases. As γ is of the order of magnitude of 10^{-2} , the first instability criterion is also fulfilled, but the second probably not. So according to the theory of DNESTROVSKY, KOSTOMAROV, and PISTUNOVICH, the electrostatic resonance instability will probably not occur. HALL, HECKROTTE, and KAMMASH²³) find as necessary condition for the instability that $d_{e\parallel}$ (= the Debye length corresponding to $kT_{e\parallel}$) must be smaller than $\frac{1}{3.4} r_{ci}$. There is also a minimum length necessary to instability:

$$\left(\frac{L}{2r_{ci}}\right)^2 \gg \pi^2 \frac{m_i T_{e\parallel}}{m_e T_{i\perp}}.$$

In HELIOS $d_{e\parallel}$ is of the order of magnitude of mm and r_{ci} of the order of magnitude of cm, so the first condition is ful-

filled. However, the length is not sufficient for these instabilities to develop. So also according to the theory of HALL, HECKROTTE, and KAMMASH, we do not expect electrostatic resonance instability.

- d) The universal (or drift) instability ²⁴⁾ may occur in a plasma with an inhomogeneous density distribution in a magnetic field. The instability can develop as well in a rarefied hot ('collisionless') plasma as in a low-temperature plasma. The instability leads to the excitation of waves that are essentially perpendicular to the magnetic field, but with a finite component along the field. The characteristic frequency is given by:

$$\omega_n = k_{\perp} \frac{cT}{eH} \frac{n'_e}{n_e} \approx k_{\perp} v_{ith} \frac{r_{ci}}{r_{pl}},$$

if $r_{pl} \approx n_e/n'_e$. In a bounded plasma with radius r_{pl} , k_{\perp} is determined by the circumference of the plasma cylinder:

$k_{\perp} = m/r_{pl}$. For $m = 1$, $k_{\perp} = 1/r_{pl}$.

The phase velocity of the oscillations coincides with the velocity of the plasma drift due to the magnetic field and the pressure gradients (drift waves). For plasmas with $1 > \beta > m_e/m_i$, the most dangerous perturbations are those with phase velocities in the range $v_{ith} \ll \omega/k_z \lesssim v_A$. The instability associated with low-frequency drift waves ($\omega \ll \omega_{ci}$) exists for an arbitrarily small density gradient.

For HELIOS: $r_{ci}/r_{pl} \approx 1/4$; $r_{pl} \approx 5$ cm; $k_z \geq 2\pi/L \approx 4 \times 10^{-2} \text{ cm}^{-1}$;

$v_{ith} \approx 5 \times 10^7$ cm/sec; $v_A \approx 10^9$ cm/sec;

$\omega_{ci} \approx 3 \times 10^7 \text{ sec}^{-1}$.

From this follows:

$\omega_n \approx 3 \times 10^6 \text{ sec}^{-1}$; thus $\omega_n \ll \omega_{ci}$;

$\omega_n/k_z \gtrsim 10^8 \text{ cm/sec}$; thus $v_{ith} \lesssim \omega/k_z \ll v_A$;

and without stabilizing effects we should expect the occurrence of the universal instability.

Stabilization of the universal instability may be obtained, if the phase velocity is of the order of the thermal velocity
force and sign $g = -\text{sign } \partial n/\partial x$. In case the B lines are convexly bend toward the plasma, $g < 0$ and stability depends approximately on the sign of $(v_{th}^2/R) - (v_{ith}^2/r_{pl})$. In those regions where R is small we may expect stability.

of the ions, so that the oscillations are damped by ion Landau damping. Thus increasing of k_z or decreasing of ω_n may bring stabilization. This may be obtained in the following three ways:

- decreasing the length ($k_z = 2\pi/L$) may bring stabilization, if $L/r_{pl} \lesssim 10$. This is difficult to obtain in HELIOS, as we have seen in section 5 that L should be larger than 2 m in order to bring in the I.C. waves.
- Shear in the magnetic field increases the wave vector along B by an amount $k_{\perp} \int (d\theta/dx) dx$. Stability may be obtained, if $r_{pl} (d\theta/dx) > 1/4 (r_{ci}/r_{pl})^{2/3}$. This condition is fulfilled in the largest part of HELIOS, if the min. B field is applied ⁺).
- Calculations by MIKHAILOVSKAYA and MIKHAILOVSKII ²⁶⁾ show that the drift instability will be stabilized at values of B greater than about 0.13, which is about the case in HELIOS.

From all this we may expect that the min. B field and the high B make HELIOS stable against universal instability.

Summarizing our considerations on the stability of HELIOS, firstly we expect the plasma to be stable against macroscopic instabilities, and secondly we do not expect to be bothered by microinstabilities either. The min. B field may not only be a remedy against interchange instability, but also against mirror instability and universal instability. The high temperature (and B) of the plasma is favourable against instabilities of the electrostatic type.

⁺) D. PFIRSCH pointed out to me that a stabilizing effect against universal instability may be expected also from the following argument: The instability criterion given in his "Habilitationsschrift" ²⁵⁾ reads:

$$g - \frac{2kT}{m_i} \left\{ \frac{1}{n} \frac{\partial n}{\partial x} + \frac{1}{T} \frac{\partial T}{\partial x} \right\} > 0, \text{ instable,}$$

where $g \simeq v_{iH}^2/R$ (R = radius of curvature) is the centrifugal force and $\text{sign } g = - \text{sign } \partial n/\partial x$. In case the B lines are convexly bend toward the plasma, $g < 0$ and stability depends approximately on the sign of $(v_{iH}^2/R) - (v_{iH}^2/r_{pl})$. In those regions where R is small we may expect stability.

In conclusion we may remark that all considerations on microinstabilities are calculated with a linearized theory, so that nothing is said about plasma losses, if they occur. Though they are possible enemies for confinement, they may be good for heating!

Note

The considerations on HELIOS given in this seminar resulted from discussions with G. Cattanei, H. Grawe, B. Oswald, G. Siller, and K. Wöhler, who are all working on the HELIOS project.

Addendum

Since the seminar on HELIOS was held, some new material on the diverse aspects which were discussed became available.

Ad 3 (Dimensions and Design)

From a discussion with W.B. Kunkel (Berkeley) it became clear that capacitive coupling of I.C. waves to the plasma (as shown in Fig. 2) is probably not suitable for heating the ions in our machine. The difficulty being that the coupling requires good electrical contact of the plasma with the electrodes, which implicitly also means a good thermal contact and thus cooling of the plasma and generation of impurities.

Although technically more difficult, the r.f. energy must be coupled to the plasma with a Stix coil. A proposal to this purpose was made by W.M. Hooke (Princeton) - and independently by K.H. Schmitter - founded on the experiences obtained from recent I.C.R. heating experiments on the C-stellarator made by YOSHIKAWA et al. ²⁷). The wall of the microwave container will not be a closed surface, but is composed of parallel strips, as shown in Fig. 4:

in a 'stuffed cusp' field (which has min. B field properties) showed that the instabilities, which are observed in the E.C.R. plasma if the neutral gas feed is stopped, can be stabilized by application of a min. B field. This result is of particular significance for the HELIOS

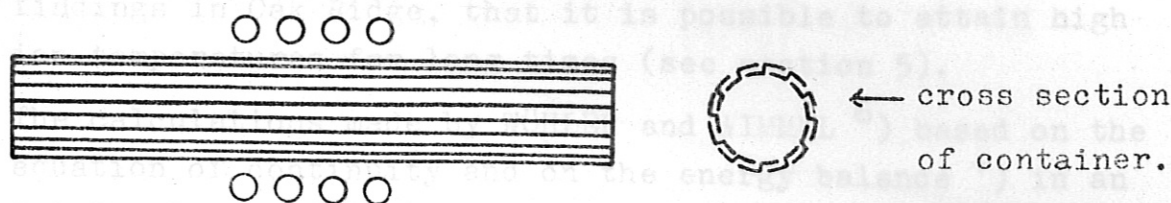


Figure 4

Microwave container surrounded by Stix coil.

Such strips were used in the C-stellarator for short-circuiting the E_z fields induced in the plasma by the r.f., thus preventing arcing between the coil sections. The strips have to be layed out in such a way that the leakage of the microwaves is as small as possible (high Q factor of the container). The Stix coil has to be in the middle part of the container and far enough from the end plates that not too much r.f. energy is dissipated there.

Ad 4 (E.C.R. Plasma)

Several suppositions made in the seminar about the E.C.R. plasma are verified by recent experiments:

- 1) The plasma density n_e increases with microwave frequency. This was observed in preliminary experiments with 8 mm-waves in Oak Ridge. The microwave power absorption was found to be 10 to 20 times larger than in a similar 3 cm experiment at the same power²⁸).
- 2) From another experiment made at Oak Ridge it was concluded that the neutral particle density in the center is reduced by a factor ≈ 10 by the plasma which surrounds the hot core of the E.C.R. plasma²⁸).
- 3) Experiments done by HARTMAN²⁹) in Livermore with a 3 cm E.C.R. plasma in a 'stuffed cusp' field (which has min. B field properties) showed that the instabilities, which are observed in the E.C.R. plasma if the neutral gas feed is stopped, can be stabilized by application of a min. B field. This result is of particular significance for the HELIOS

vector energy loss.

project, as it indicates, together with the above mentioned findings in Oak Ridge, that it is possible to attain high ion temperatures for long times (see section 5).

- 4) The calculations made by WÖHLER and WIMMEL⁶⁾ based on the equation of continuity and on the energy balance⁺) in an E.C.R. plasma, indicated that a minimum power density of the microwaves is required (for hydrogen ≥ 0.25 kW/l), if all the electrons are to be heated to temperatures $T_e \geq 200$ eV.

For simplicity it was assumed that the plasma occupies the whole volume of the container, that the total number density of neutrals and electrons ($n_o + n_e$) is constant, and that all electrons acquire the same kinetic energy. As initial conditions were chosen $n_e/(n_o + n_e)(t=0) = 10^{-2}$ and $T_e(t=0) = 13$ eV.

Some of the results are shown in Fig. 5, where electron temperature T_e and degree of ionization are drawn as functions of a normalized time

$$\frac{(n_o + n_e)}{10^{12}} \times t$$

for two values of normalized power density

$$\left(\frac{10^{12}}{n_o + n_e}\right)^2 \times \frac{P}{V}$$

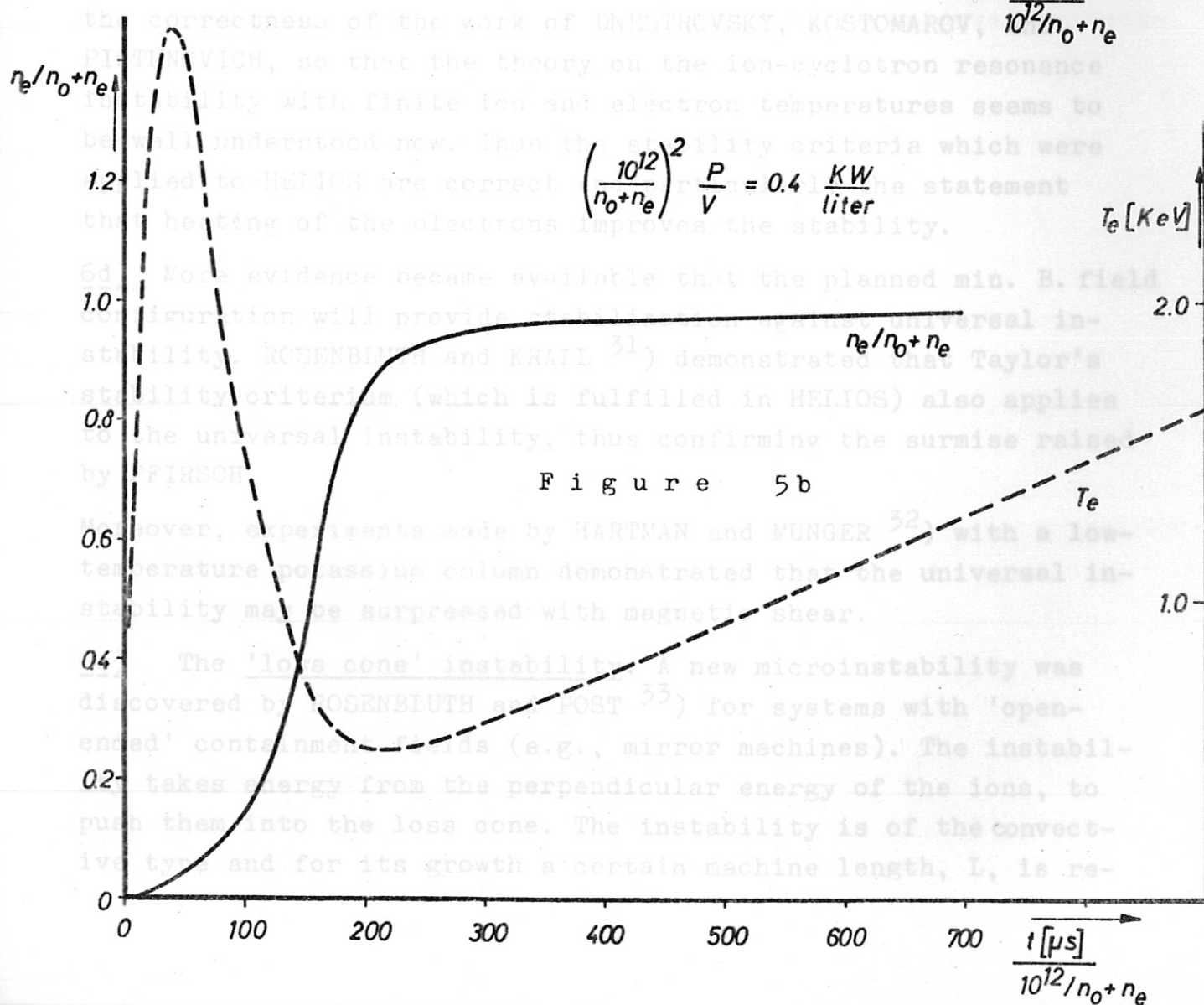
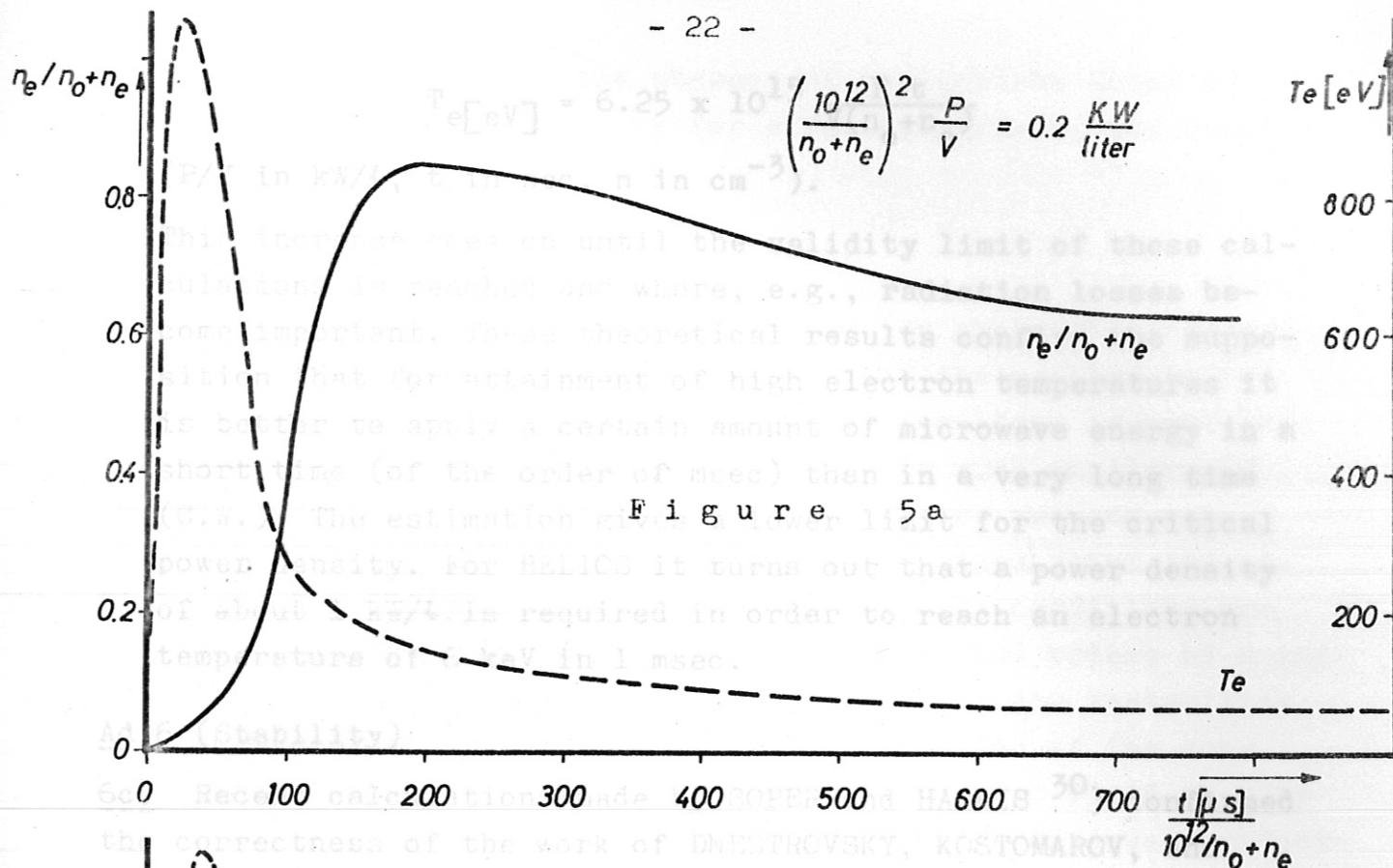
(respectively 0.2 and 0.4 kW/l).

It follows:

For subcritical power densities ($P/V \lesssim 0.25$ kW/l) the density and the temperature reach a stationary value after about 0.5 msec, where $T_e(t \rightarrow \infty) \lesssim 100$ eV.

For supercritical power densities 'full ionization' is attained after about 0.3 msec, and practically all the microwave energy is used for heating the electrons. The electron temperature raises linearly with time according to the relation:

⁺) The production rate of the electrons ($\frac{dn_e}{dt}$) is given by the ionization rate minus the loss rate out of the mirror. The power flow to the plasma ($\frac{d(n_e T_e)}{dt}$) is given by the microwave power density, the energy loss by ionization and the convective energy loss.



$$T_e[\text{eV}] = 6.25 \times 10^{15} \frac{P t}{V(n_o + n_e)}$$

(P/V in kW/l, t in sec, n in cm⁻³).

This increase goes on until the validity limit of these calculations is reached and where, e.g., radiation losses become important. These theoretical results confirm the supposition that for attainment of high electron temperatures it is better to apply a certain amount of microwave energy in a short time (of the order of msec) than in a very long time (C.W.). The estimation gives a lower limit for the critical power density. For HELIOS it turns out that a power density of about 1 kW/l is required in order to reach an electron temperature of 6 keV in 1 msec.

Ad 6 (Stability)

6c) Recent calculations made by SOPER and HARRIS ³⁰⁾ confirmed the correctness of the work of DNESTROVSKY, KOSTOMAROV, and PISTUNOVICH, so that the theory on the ion-cyclotron resonance instability with finite ion and electron temperatures seems to be well understood now. Thus the stability criteria which were applied to HELIOS are correct and particularly the statement that heating of the electrons improves the stability.

6d) More evidence became available that the planned min. B. field configuration will provide stabilization against universal instability. ROSENBLUTH and KRALL ³¹⁾ demonstrated that Taylor's stability criterium (which is fulfilled in HELIOS) also applies to the universal instability, thus confirming the surmise raised by PFIRSCH.

Moreover, experiments made by HARTMAN and MUNGER ³²⁾ with a low-temperature potassium column demonstrated that the universal instability may be suppressed with magnetic shear.

6e) The 'loss cone' instability. A new microinstability was discovered by ROSENBLUTH and POST ³³⁾ for systems with 'open-ended' containment fields (e.g., mirror machines). The instability takes energy from the perpendicular energy of the ions, to push them into the loss cone. The instability is of the convective type and for its growth a certain machine length, L, is re-

quired. Assuming that in the absence of reflections about 10 to 20 e-foldings are necessary for effective growth, the stability criterium becomes:

$$L < 10^4 \sqrt{m_e/m_i} r_{ci}, \text{ stable.}$$

In the final heating stage of HELIOS the ion cyclotron radius r_{ci} is of the order of magnitude of cm, so that the machine is not long enough for appreciable growth of the loss-cone instability. Before the I.C.R. heating is applied, r_{ci} is much smaller and instability may occur, though its growth may be reduced considerably for $T_e/T_i \gg 1$. Another question arises about the efficiency of reflections. If reflecting boundary conditions are assumed, the predicted critical length is several orders of magnitude smaller. It seems likely that the onset of the instability can be influenced by changes in the configuration of the confining field. As in the HELIOS design the magnetic field is very flexible, this loss-cone instability could be studied very effectively.

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